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A NEW APPROACH TO THE THEORY OF HIERARCHIC WAVE-OSCILLATION DYNAMIC SYSTEMS

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Abstract. *A set of new ideas and concepts (which the authors treat as a new version of the theory of hierarchic dynamic systems) are set forth in the article. In turn, the set of hierarchic principles is put in the methodogical basis of this theory. It is shown, a peculiar version of the theory of hierarchic waves and oscillations can be developed on the basis of proposed general theory.*

I. INTRODUCTION

As it is well known, a conventional version of the theory of hierarchic dynamic systems (see [1-5]) is most popular now. Another (new) version of the hierarchic approach has been proposed in monographs [6-9] and papers [10-13]. When a proper set of dynamical variables for each hierarchic level is introduced, therein, the proper variables are chosen in such a way that the mathematical structure of the dynamic equations for every hierarchic level (in these new variables) is the same.

Hence, essentially new basic concepts are used in the proposed version of hierarchic theory. Further development of such theory is undertaked in this work. The main attention is given to the nonlinear wave-oscillation dynamic systems with explicitly expressed hierarchy of periods (frequencies) of oscillations.

Let us begin with discussion of the set of hierarchic principles that are put into basis of the theory considered. These principles [7-13] originate from relevant generalization of totality of experimental facts and observations well known today.

II. HIERARCHIC PRINCIPLES

The five following hierarchic principles form a basis of the discussed version of hierarchic theory, e.g. one general hierarchic principle and four particular fundamental ones.

General hierarchic principle. *Everything in the Universe has the hierarchic nature.*

The other four principles concern the structure, general dynamic, informative, and thermodynamic features of the hierarchic systems.

The principle of information compression: *each higher hierarchic level is always simpler than the preceding one* (the concept of (algorithmic) complexity we determine as a minimal length of corresponding algorithm describing the considered system.

The principle of hierarchic resemblance (self-modeling principle or holographic principle): *each hierarchic level in its general properties represents the system as a whole*

The hierarchic analogue of the second thermodynamic principle: each higher hierarchic level has less information entropy than the preceding one. The information about the state of the considered dynamic system is directed from lower hierarchic levels to higher ones, whereas the information in the form of controlling action is directed oppositely always, i.e., the controlling information is always directed from higher to lower.

Hierarchic analogue of the third thermodynamic principle: the highest level of a closed hierarchic system is characterized by vanishing information entropy. Using this observation a number of methods for quantitative analysis of such systems (hierarchic methods) were worked on successfully [7-13].

III. DYNAMIC EQUATION OF THE ZEROTH HIERARCHIC LEVEL

Then let us to set factorization (i.e., mathematical description) the above formulated hierarchic principles. For simplicity we assume that some oscillation dynamical system can be described by the following vector exact differential equation

$$\frac{dz}{dt} = Z\left(z,t\right), \tag{1}$$

where $z = \{z_1, z_2, ..., z_n\}$ is some vector and $Z = \{Z_1, Z_2, ..., Z_n\}$ is the vector function in a $n-$dimensional Euclidean space $\mathbf{R}_n$, $t\in [0,\infty]$ (t is the laboratory time, for example). The initial conditions are standard:

$$z\left(t=0\right) = z_0. \tag{2}$$

Discussing properties of the vector function Z and the initial conditions, we limit ourselves by separation only three most characteristic situations:
the determined hierarchic system, i.e. the function Z satisfies the uniqueness of the solution of corresponded equation and the initial conditions are given uniquely;
the stochastic hierarchic systems that are characterized by random initial conditions;
the chaotic systems with properties caused by nonlinear nature of the system and other causes.

Then we take into account the general hierarchic principle, i.e., we assume that our dynamic system possesses a hierarchic structure. According to the principle of hierarchic resemblance, each hierarchic level of the system should be described by an equation which resembles mathematical structure. Hence, the main hierarchic problem is elaboration of special procedures for hierarchic transformation of the initial equation (1) to each level. Therein, this procedure should satisfy all requirements that follows from the above formulated hierarchic levels.

Thus, the first step in the discussed direction is the hierarchization of the considered system. For this we separate all oscillation periods (the evident as well as the hidden ones) and organize their in the hierarchical manner. Or, in other words, we accept the oscillation

periods as characteristic times for the hierarchic levels. Dividing all periods on the period of highest level we obtain a possibility to construct the following strong hierarchic series:

$$\varepsilon_1 << \varepsilon_2 << .... << \varepsilon_\kappa << .... << \varepsilon_m << 1. \tag{3}$$

This series we will use later as a basis for constructing the looking for procedure of hierarchization. Namely, we use the parameters ε_κ as small parameters for corresponding asymptotic expansions. But, the question arises: how to calibrate the system parameters in the scale of the chosen small parameters ε_κ. We use for this the above formulated hierarchic principles. Let us begin with the hierarchic analogy of the second and third thermodynamic principles. One can prove the latter affirmation equivalent to the assertion that *each higher hierarchic level has less characteristic velocities of the varying of dynamic variables than the preceding one.* The affirmation that characteristic velocities of dynamical variables varying of the highest level of the closed hierarchic system are zero is the consequence of the fourth hierarchic principle. Using these suppositions, we formulate the *general* definition of the hierarchic scale parameter of a problem ε_κ for the κ-th level through the variables of this level:

$$\varepsilon_\kappa \sim \left|\frac{dz^{(\kappa+1)}}{dt}\right| \Big/ \left|\frac{dz^{(\kappa)}}{dt}\right| << 1, \tag{4}$$

where $z(\kappa)$ is the vector of the proper variables of a κ-th hierarchic level. In turn, this gives a possibility to treat the initial dynamic equation (1) as the *zero-level-equation*:

$$\frac{dz^{(0)}}{dt} = Z^{(0)}\left(z^{(0)}, t, \varepsilon_1, ..., \varepsilon_m\right), \tag{5}$$

Further more , we expand the right part of the equation (5) in a power series with respect to small parameters ε_κ. Because hierarchy here is expressed strongly (see (3), (4)), we can temporarily neglect the hierarchic structure with levels higher the first one:

$$\frac{dz^{(0)}}{dt} = Z^{(0)}\left(z^{(0)}, t, \varepsilon_1\right), \quad z^{(0)}\left(t_0\right) = z^{(0)}, \tag{6}$$

where $\varepsilon_1 << 1$, in accordance the above introduced designations, is the scale parameter for the first hierarchic level. The physical sense of the approximation is that only influences of neighboring hierarchic levels are accounted.

This calculation situation is illustrates in Fig. 1. The illustration of the scheme of interactions between the zeroth and other hierarchic levels of the dynamic system is given. Taking into consideration the above-formulated supposition about accounting only neighboring interactions only, we obtain the system where interaction is taken between neighboring levels only. This approximation allows us to simplify calculation procedure essentially.

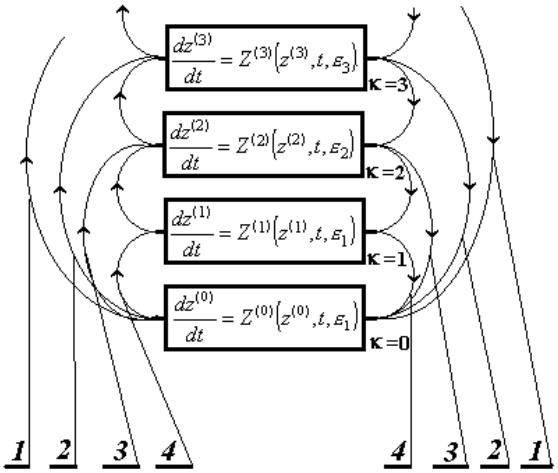

Fig. 1. Illustration of the scheme of interactions between the zeroth and other hierarchic levels of a dynamic system.

IV. DYNAMIC EQUATION FOR HIGHER HIERARCHIC LEVELS: STRUCTURAL AND FUNCTIONAL (DYNAMIC) OPERATORS

Now we introduce the concepts of the structural and functional (dynamic) operators which are responsible for the structural and functional (dynamic) hierarchies correspondingly. To describe structural hierarchy, one introduces structural operator $M^{(\kappa)}$

$$Z^{(\kappa+1)}\left(z^{(\kappa+1)}, t\right) = M^{(\kappa)} Z^{(\kappa)}\left(z^{(\kappa)}, t\right), \tag{7}$$

where $Z^{(\kappa)}$ is the vector function defined as the right part of relevant standard dynamic equation (of the type (6) but for the for κ-th hierarchic level only).

Thus, the structural operator $M^{(\kappa)}$ is responsible for the hierarchic resemblance in the discussed dynamic system. To obtain the functions $Z^{(\kappa+1)}\left(z^{(\kappa+1)}, t\right)$ (for the known functions $Z^{(\kappa)}\left(z^{(\kappa)}, t\right)$) various integral transformations are used. In this cycle of works we discuss only integral operators, namely the averaging and Fourier transformations, correspondingly.

The transformation functional (dynamic) operator $\hat{U}$ (a function in the case of integral structural operators) describes the dynamics of connections between proper variables of two neighboring hierarchic levels. We define the transformation functional (dynamic) operator as

$$z^{(\kappa)} = \hat{U}^{(\kappa+1)}\left(z^{(\kappa+1)}, t\right). \tag{8}$$

As one can be convinced, the introduced operators (structural (7) and functional (8)) satisfy also other hierarchic principles.

Let us now discuss some characteristic features of their application for the theory of hierarchic oscillations in electrodynamic systems. Therein, here, in this article, we will take an interest, mainly, in the mathematical part of the problem.

V. FORMULATION OF THE HIERARCHIC SINGLE-PARTICLE ELECTRODYNAMIC PROBLEM

We start this section with discussion of some general features of the hierarchic version of classical single-particle electrodynamic problems. First of all we accept that the acting electromagnetic field can be described as the superposition of the wave and non-wave parts, correspondingly and the latter (non-wave part) is assumed slowly varying, non-periodic quasi-stationary and quasi-homogeneous fields. This means that arrangement of the electric and magnetic fields acting on the charged particles can be represented in the form:

$$\vec{E} = \frac{1}{2}\sum_{l}\left[\vec{E}_l \exp\left\{im_l p_l\right\} + c.c.\right] + \vec{E}_0; \quad \vec{B} = \frac{1}{2}\sum_{j}\left[\vec{B}_j \exp\left\{im_j p_j\right\} + c.c.\right] + \vec{B}_0, \quad (9)$$

where $\vec{E}$ and $\vec{B}$ are electric field intensity and magnetic induction field. $\vec{E}_l = \vec{E}_l(\vec{r}, t)$ and $\vec{B}_j = \vec{B}_j(\vec{r}, t)$ are slowly varying complex amplitudes of electric and magnetic induction fields of partial waves (proper and stimulated); $\vec{E}_0 = \vec{E}_0(\vec{r}, t)$, $\vec{B}_0 = \vec{B}_0(\vec{r}, t)$ are slowly varying quasi-stationary and quasi-homogeneous electric and magnetic induction fields, and m_l, m_j are wave harmonic numbers (m_j, $m_l = \pm 1, \pm 2, ...$). The quantities

$$p_q = \omega_q t - \vec{k}_q \vec{r}, \quad q = j, l \quad (10)$$

are phases of oscillations, ω_q is the angular frequency, k_q is the wave vector, t is the laboratory time and $\vec{r}$ is the position vector. In the case of proper waves we have the dispersion relations between ω_q and k_q $\left(\vec{k}_q = \vec{k}_q(\omega_q)\right)$.

In terms of the above discussed general hierarchic theory, assigning form (10) to the fields determines form of the chosen structural operator $M^{(\kappa)}$.

In the framework of considered theory electron, ion or plasma beams of two extreme types (low- and high-intensity beams, respectively) could be described. The characteristic feature of beams of the first type is that intensity of Coulomb interaction between the charged particles is relatively low. In fact, such beams are just streams of nearly independent drifting charged particles. Hence, we have a possibility to study of a separate particle interaction with the fields. The single-particle theory of motion of charged particles in electromagnetic fields is a result of such study. In turn, the single-particle problem is the main point of this theory. The particular case of the multi-particle problem can be realized here by simply summing up all separate single-particle interactions.

In contrast, the particle-particle Coulomb interactions can be rather intense in the high-intensity beams. At sufficiently high densities the beams show a collective behavior and they treated as a quasi-continuous flow of drifting charged (or quasi-neutral) plasmas. Hence, in this case the charged particle beam should be regarded as a whole electrodynamic object.

It is obvious that owing to the above-discussed differences of the beams of both types the mathematical descriptions of these objects should also be different. Thus, beam's motion of the first type (rarified plasma beams) can be considered as the motion of aggregate of individual particles (Lagrange formalism). In this case, the beam motion can be

described by the single-particle equations in the Hamiltonian form:

$$\frac{dH_\alpha}{dt}=\frac{\partial H_\alpha}{\partial t};\quad \frac{d\vec{P}_\alpha}{dt}=-\frac{\partial H_\alpha}{\partial t};\quad \frac{d\vec{r}}{dt}=\frac{\partial H_\alpha}{\partial \vec{P}_\alpha}, \tag{11}$$

or by the Lorenz equation:

$$\frac{d\vec{\beta}_\alpha}{dt}=\frac{q_\alpha}{m_\alpha\gamma_\alpha c}\left\{\vec{E}+\left[\vec{\beta}_\alpha\vec{B}\right]-\vec{\beta}_\alpha\left(\vec{\beta}_\alpha\vec{E}\right)\right\}. \tag{12}$$

Here: $H_\alpha=\sqrt{m_\alpha^2c^4+c^2\left(\vec{P}_\alpha-\frac{q_\alpha}{c}\vec{A}\right)}+q_\alpha\varphi$ is the Hamiltonian of a charged particle of type α; q_α is its charge ($q_\alpha=-e$ for electron and $q_\alpha=+Ze$ for an ion, e is electron charge, Z is charge number of the ion), mα is the rest mass of particle of sort α; $\vec{A}$ is vector potential and φ is scalar potential of the electromagnetic fields [4]; $\vec{\beta}_\alpha=\vec{v}_\alpha/c$ is the non-dimension particle velocity, (here $\vec{v}_\alpha$is the velocity, cis light velocity in vacuum) $\vec{P}_\alpha$ is the canonical momentum, $\vec{r}_\alpha$ is the position vector of particle α; $\gamma_\alpha=\left(1-\beta_\alpha^2\right)^{-1/2}$ is the relativistic factor (here $\beta_\alpha=\left|\vec{\beta}_\alpha\right|$).

The hierarchic part of the problem consists of two main stages. The first stage is the reducing of the equations (11) or (12) to one of so-called standard forms. The second one is the asymptotic integration of the standard equations. Here we accept such hierarchic standard form as the system with fast rotating phases:

$$\begin{aligned}
\frac{dx}{dt}&=X\left(x,\psi_1,...,\psi_\kappa,...,\psi_m,\xi_1,...,\xi_m\right);\\
\frac{d\psi_1}{dt}&=\xi_1\omega_1\left(x\right)+Y_1\left(x,\psi_1,...,\psi_\kappa,...,\psi_m,\xi_1,...,\xi_m\right);\\
&.\ .\ .\ .\ .\ .\ .\ .\\
\frac{d\psi_m}{dt}&=\xi_m\omega_m\left(x\right)+Y_m\left(x,\psi_1,...,\psi_\kappa,...,\psi_m,\xi_1,...,\xi_m\right),
\end{aligned} \tag{13}$$

where scale parameters $\xi_\kappa=1/\varepsilon_\kappa$ are the relevant terms of the strong hierarchic series (see for comparison (3)):

$$\xi_1>>\xi_2>>...>>\xi_\kappa>>...>>\xi_m>>1, \tag{14}$$

and x is the vector whose components are slowly varying variables only, ψ_κ are partial vectors of different hierarchy of the vector of fast rotation (revolving) phases ψ, ω_κ are slowly varying part of velocity vector of fast rotating phases, Y_κ are relevant vector functions, κ is hierarchic level number, and m is the number of the highest hierarchic level.

Within context of the definition (4), large hierarchic parameters ξ_κ in (13), (14) can be determined as

$$\xi_\kappa\sim\left|\frac{d\psi_{\kappa l}}{dt}\right|\Big/\left|\frac{dx_q}{dt}\right|, \tag{15}$$

where $\psi_{\kappa l}$ is l-th component of vector ψ_κ and x_q is q-th component of vector x. Here we consider the rate of variation of x_q not exceeding rates of variations of other slow variables and the component $\psi_{\kappa l}$ is the "slowest" of other fast components of the vector ψ_κ. Thus,

with general definition of hierarchic scale parameter (4) we affirm that variables $\psi_{\kappa l}$ (in the case $\kappa = 1$) and x_q represent two neighboring different hierarchic levels.

VI. CLASSIFICATION OF OSCILLATORY PHASES AND RESONANCES. HIERARCHIC TREE

Accordingly with the general principles of the hierarchic approach we should reduce initial systems (11) or (12) to the standard form (13). Therefore, the first step consists in determining all elements of "slow vector" x and "fast phase vector" ψ in (13). In other words, it consists in classifying total set of variables of systems (11) and (12) as slow and fast ones, respectively.

The Hamiltonian H and variables $\vec{P}$, $\vec{\beta}$ (or $\vec{v} = \vec{\beta}c$) and $\vec{r}$, as a rule, might be considered as slow variables. In contrast, classification of the Lagrange phases of particle oscillations is more complicated. In this case, we should separately find the set of phases forming component basis of vectors Φ and x. Besides that, we must divide rotating phases of particle oscillations into those associated with explicit and hidden periods of oscillation. The phases of the first group are related with phases of the particle oscillations in the wave-type fields. The periodicity of these fields is responsible for the periodic character of electron motion with respect to the same wave phases (10)

$$p_q = \omega_q t - \vec{k}_q \vec{r}. \tag{16}$$

The second case is much more complicated. It can be associated with phases of periodic particle motion in non-periodical quasi-stationary intrinsic beam fields, external focusing and beam-forming fields, etc. The procedure separating hidden periods of arbitrary functions is known in mathematics [6-8, 14-22]. There are also semi-empirical approaches based on qualitative peculiarities of a studied physical picture [6-8].

Analyzing the rates of particle phase varying we find that their total set involves both types of phases: slow components of vector x and fast components of vector ψ. Moreover, some nonlinear combinations of two, three or more Lagrange fast phases produce slowly varying functions. Each slow phase resulting from this "combination" action (m-fold, in general case) corresponds to some physical mechanism of particle resonance. We distinguish quasi-linear and parametric resonance. Quasi-linear resonances are characterized by two-phase (or m-phase, in general case) combinations where one of phases only is associated with explicit period of the system. The magnitude of stimulated wave force acting on the particle is always linearly dependent on wave field amplitude in the lowest order in some small parameter. Therefore, these resonances are quasi-linear ones. Examples: cyclotron resonance, various types of synchrotron resonances, etc.

Parametric one-particle resonances correspond to cases when all phases forming slow phase combination are of wave nature. For instance, parametric resonances of third-, fourth-, and higher orders [6-8] can realize in free electron lasers (FELs), in parametric electronic devices (Adler's lamp, parametric electron-wave lamps), etc. [6-8]. In general, slow $\theta_{\nu g}$ and fast $\psi_{\nu g}$ combined phases of coupled-pair parametric or quasi-linear

resonances can be defined as

$$\begin{aligned} \theta_{\nu g} &= \frac{m_\nu n_\nu}{m_g n_g} p_\nu - \sigma_{\nu g} p_g; \\ \psi_{\nu g} &= \frac{m_\nu n_\nu}{m_g n_g} p_\nu + \sigma_{\nu g} p_g. \end{aligned} \tag{17}$$

where $\sigma_{\nu g} = \pm 1$ are sign functions, m_ν and m_g are numbers of wave field harmonics, and n_ν and n_g are numbers of electron oscillation harmonics in these fields. Hence, the lowest order of relevant amplitudes of stimulated waves in the case of parametric resonances is quadratic. Therefore, we classify parametric resonances as nonlinear ones.

According to the above discussed hierarchic principles we define components of the scale parameter tensor according to reciprocal rate of varying of fast to slow combined phases (17)

$$\xi_{\nu g r k} \sim \left| \frac{d\psi_{\nu g}}{dt} \right| \Big/ \left| \frac{d\theta_{\nu g}}{dt} \right| >> 1. \tag{18}$$

It means that slow $\theta_{\nu g}$ and fast $\psi_{\nu g}$ combined phases are variables of different hierarchic levels. Therein, we assume the rates of change of velocities are of the same order (in magnitude) as rates of velocities of other slow variables (see (15)). Condition (18) is one of key relationships of the proposed theory of hierarchic wave and oscillations. It has the physical meaning of generalized hierarchic resonances in the system. Indeed, here the main resonances as well as the resonances on harmonics are taken into account.

Then we analogously take into account the non-resonant phases of oscillations. The only difference is that instead of slow and fast combined phases we choose ordinary slow and fast phases. On the next stage of calculation, we construct hierarchic series of components of both tensors of scale parameters in the form (14). Hierarchic series (14) are the essential point of the theory called the theory of hierarchic oscillations. In its physical nature this series follows from the above-accepted assumption on influences between neighboring hierarchic levels only (see Fig. 1 and corresponding comments).

Relevant hierarchic trees can be constructed for the discussed oscillatory hierarchic systems. For the sake of simplicity we consider that only simple (two-fold) resonances can be realized in the system. Or, in other words, that each two structural elements (fast phases) of lower hierarchic level form one structural element (slow phase) of the next higher level. Therefore, phase-resonant hierarchic tree can be constructed (Fig. 2).

Then, we discuss the hierarchic tree (9), from point of view of the above-formulated hierarchic principles. We see that a few "κ-level phases" (they are two phases only in this particular case) forms each combined phase of the next $(\kappa + 1)$-level. This means that the higher hierarchic level the less number of oscillative phases are. The latter can be considered as obvious illustration of the compression hierarchic principle.

The hierarchic resemblance principle holds due to specific mathematical construction of equations for each hierarchic level. As it is noted above, all these equations are similar. At that, characteristic velocities of changing of variables of different levels are higher the lesser number of hierarchic level is. It is readily seen that any oscillations are not on the highest hierarchic level: $\theta_m = const$, $d\theta_m/dt = 0$, $\kappa = m$. It means that on the highest

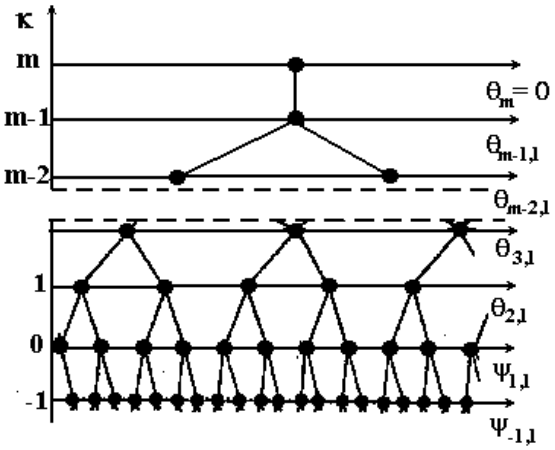

Fig. 2. The simplest case of hierarchic tree consisting of rotating (revolving) charged particle phases. Here: κ is hierarchic number of the level, $\psi_{\kappa l}$ and $\theta_{\kappa l}$ are rotating phases with rate of rotation velocities of different hierarchic order (they are shown by dark small circles), m is the highest hierarchic level.

hierarchic level information entropy is equal zero. Hence, hierarchic analogy with the third thermodynamic principle takes place here, too, and so forth.

The main idea therein consists in reducing corresponding nonlinear equations with partial derivatives (using, for instance, the concept of characteristics or some other similar) and then to use the above described single-particle algorithms. In detail calculation technologies of such a kind are described in the monographs [6-8]. Corresponding examples are given also in other our articles of this series.

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